

**MATH 2050C Mathematical Analysis I**  
**2019-20 Term 2**  
**Problem Set 12**

*due on May 1, 2020 (Friday) at 11:59PM*

**Instructions:** You are allowed to discuss with your classmates or seek help from the TAs but you are required to write/type up your own solutions. please do NOT come to campus to submit your completed assignments. Instead, you can either type up your assignment or scan a copy of your written assignment into ONE PDF file and submit through CUHK Blackboard on/before the due date. Please remember to write down your name and student ID. You can refer to the webpage under "Useful Links" below about how to submit assignments through Blackboard. **No late homework will be accepted.** All the exercises below are taken from the textbook.

**Required Readings:** Chapter 5.4

**Optional Readings:** Chapter 5.6

**Problems to hand in**

Section 5.4: Exercise # 2, 7, 8, 10

**Suggested Exercises**

Section 5.4: Exercise # 1, 3, 4, 5, 6, 9, 11, 12, 14, 15

**Challenging Exercises (optional)**

1. Section 5.4: Exercise # 13, 16
2. Let  $f : \mathbb{Q} \rightarrow \mathbb{R}$  be a uniformly continuous function. Prove that there exists a continuous function  $g : \mathbb{R} \rightarrow \mathbb{R}$  such that  $g(x) = f(x)$  for all  $x \in \mathbb{Q}$ .
3. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a function such that  $|f(x)| \leq 1$  for all  $x \in \mathbb{R}$ . Suppose  $g : [-1, 1] \rightarrow \mathbb{R}$  is a continuous injective function. Prove that  $f$  is (uniformly) continuous if  $g \circ f$  is (uniformly) continuous.